(1) $1.4(x+1) = (x+10)$

$$\frac{14x+14}{10} = x+10$$

$$14x+14 = 10x+100$$

$$\therefore \quad 4x = 86$$

$$\therefore \quad x = 21.5$$

---

(2) $1.5(x+4) = x+12$

$$\therefore \quad \frac{15x+60}{10} = x+12$$

$$\therefore \quad 15x+60 = 10x+120$$

$$\therefore \quad 5x = 60$$

$$\therefore \quad x = 12$$

---

(3) $1.6(x+1) = x+15$

$$\therefore \quad \frac{16x+16}{10} = x+15$$

$$\therefore \quad 16x+16 = 10x+150$$

$$\therefore \quad 6x = 134$$

$$\therefore \quad x = 22\tfrac{1}{3}$$

④ $(-8,-6)$ is a point where $x=-8$, $y=-6$.

Gradient $m = \frac{3}{4}$.

$$y = mx + c$$

$$\therefore c = y - mx$$

$$= -6 - \left(\frac{3}{4}\right)(-8)$$

$$= -6 + 6$$

$$= 0$$

So the co-ordinates where it crosses the y axis is $(0,0)$.

---

⑤ $(3,6)$ is a point where $x=3$ and $y=6$.

Gradient $= \frac{5}{4}$.

Work out where the point crosses the y axis:

$$y = mx + c$$

$$\therefore c = y - mx$$

$$= 6 - \frac{5}{4}(3)$$

$$= 6 - 3\tfrac{3}{4}$$

$$= 2\tfrac{1}{4}$$

$$= \frac{9}{4}.$$

So the line crosses the y axis at $\left(0, \frac{9}{4}\right)$.

Work out where it crosses the x axis:

$$y = mx + c$$

$$\therefore mx = y - c$$

$$\therefore x = \frac{y-c}{m}$$

$$= \frac{0 - \frac{9}{4}}{\frac{5}{4}}$$

$$-\frac{9}{4} \div \frac{5}{4} = -\frac{9}{4} \times \frac{4}{5}$$

$$= -\frac{9}{5}$$

So the coordinates are $\left(-\frac{9}{5}, 0\right)$

(6) At $(-5,-4)$, $x=-5$, $y=-4$.

Gradient $m = \frac{15}{8}$.

To find the y intercept and hence the co-ordinates of where the line crosses the y axis:

$y = mx + c$

$\therefore c = y - mx$

$= -4 - \left(\frac{15}{8}\right)(-5)$

$= -4 + \frac{75}{8}$

$= 9\frac{3}{8} - 4$

$= 5\frac{3}{8}$

$= \frac{43}{8}$

So the point at which the line crosses the y axis is: $\left(0, 5\frac{3}{8}\right)$

To find where the line crosses the x-axis:

$y = mx + c$

$\therefore mx = y - c$

$\therefore x = \frac{y-c}{m}$

$= \frac{0 - \frac{43}{8}}{\frac{15}{8}}$

$x = -\frac{43}{8} \div \frac{15}{8}$

$= -\frac{43}{8} \times \frac{8}{15}$

$= -\frac{43}{15}$

$= -2\frac{13}{15}$

So the x axis is crossed at the co-ordinates: $\left(-2\frac{13}{15}, 0\right)$

$-3 \leq x < 2$

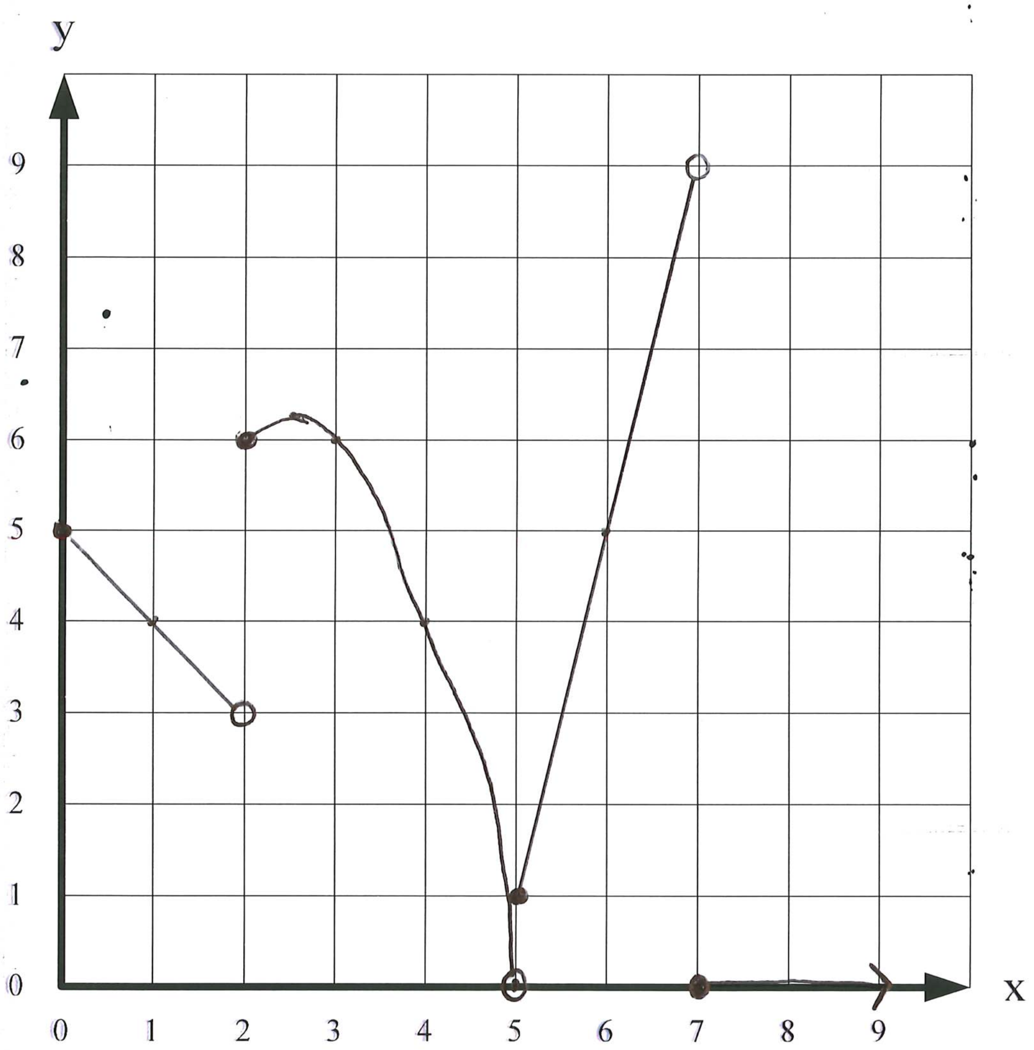
7 $\quad f(x) = 5 - x \qquad\qquad 0 \leq x < 2$

$\qquad = 5x - x^2 \qquad\qquad 2 \leq x < 5$

$\qquad = 4x - 19 \qquad\qquad 5 \leq x < 7$

$\qquad = 0 \qquad\qquad x \geq 7$

On the grid, draw the graph of $y = f(x)$



x	0	1	2	3	4	5	6	7	8	9
$5-x$	5	4	3							
$5x-x^2$			6	6	4	0				
$4x-19$						1	5	9		
0								0	0	0

2·5 | 6·25

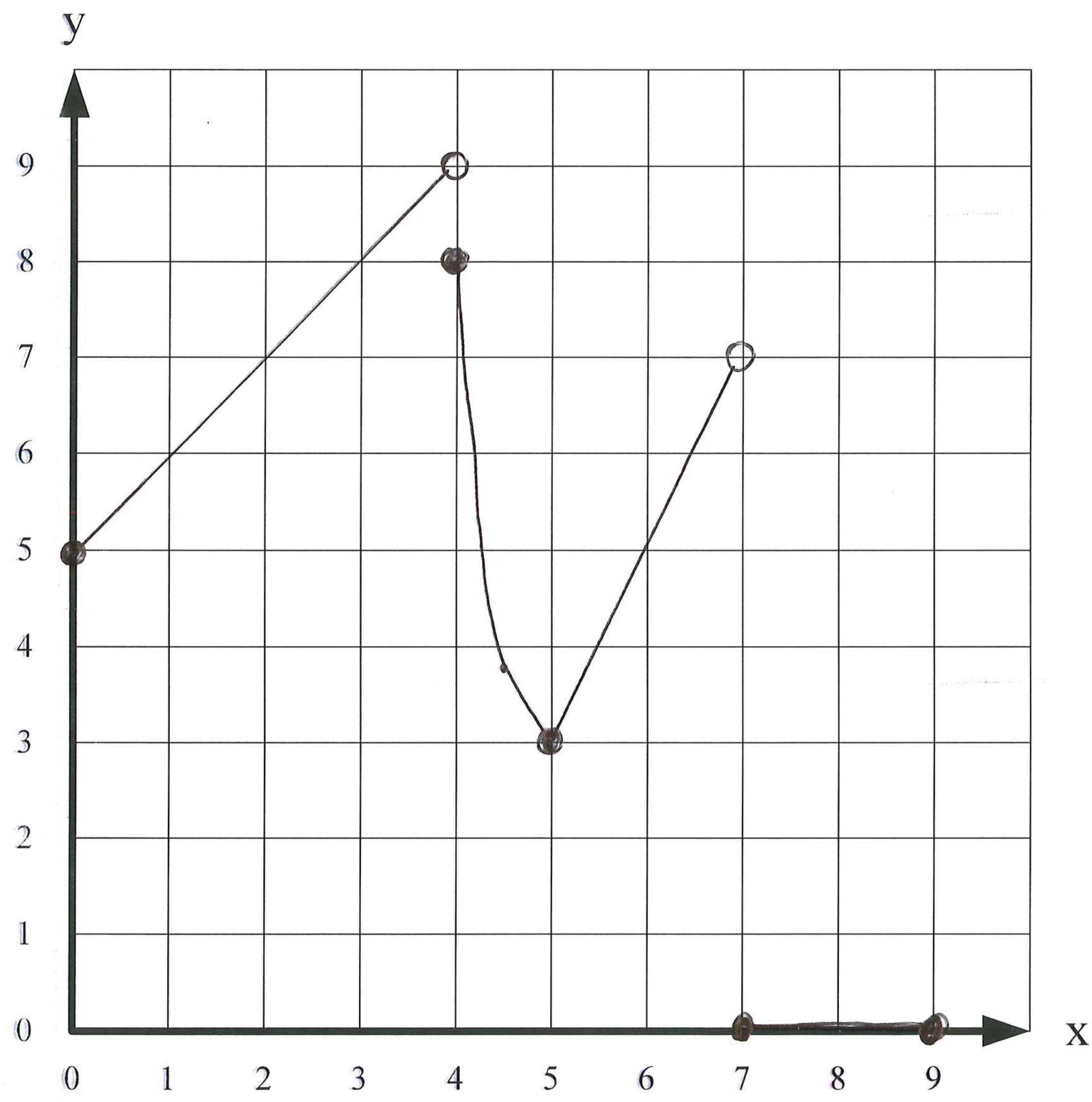
8 $f(x) = 5 + x$ $\quad 0 \le x < 4$

$\quad = 4x - x^2 + 8$ $\quad 4 \le x < 5$

$\quad = 2x - 7$ $\quad 5 \le x < 7$

$\quad = 0$ $\quad x \ge 7$

On the grid, draw the graph of $y = f(x)$



4.5

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	5	6	7	8	9	3	5	7		
					8	3		0	0	0

~~4.5~~

3.75

(10) $k(x) = 12 - 4x$

$\therefore \quad y = 12 - 4x$

$x = 12 - 4y$

Swap x and y round

$4y = 12 - x$

$y = \dfrac{12 - x}{4}$

$\therefore \quad k^{-1}(x) = \dfrac{12 - x}{4}$

---

(11) $k(x) = \dfrac{5}{3x} - 2$

$\therefore \quad y = \dfrac{5}{3x} - 2$

Swap x and y

$x = \dfrac{5}{3y} - 2$

$3y(x+2) = 5$

$3y = \dfrac{5}{x+2}$

$\therefore \quad y = \dfrac{5}{3x+6}$

$\therefore \quad k^{-1}(x) = \dfrac{5}{3x+6}$

---

(12) $k(x) = 3x + 5ax - 12$

$\therefore \quad y = 3x + 5ax - 12$

$\therefore \quad x = 3y + 5ay - 12$

$\therefore \quad x + 12 = 3y + 5ay$

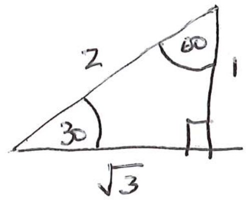
$x + 12 = y(3 + 5a)$

$\dfrac{x + 12}{3 + 5a} = y$

$\therefore \quad y = \dfrac{x + 12}{3 + 5a}$

$\therefore \quad k^{-1}(x) = \dfrac{x + 12}{3 + 5a}$

(13) $\tan^2(30^\circ) = \left(\tan(30)\right)^2$



$\tan(30) = \frac{1}{\sqrt{3}}$

$\therefore \tan^2(30) = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$

S O H
C A H
T O A

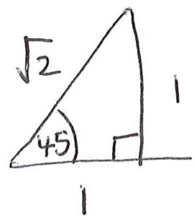
---

(14) $\sin^2(30) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

(15) $\cos^2(30) = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}$

---

(16) $\cos^2(45) = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$



---

(17) $\cos^2(60) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

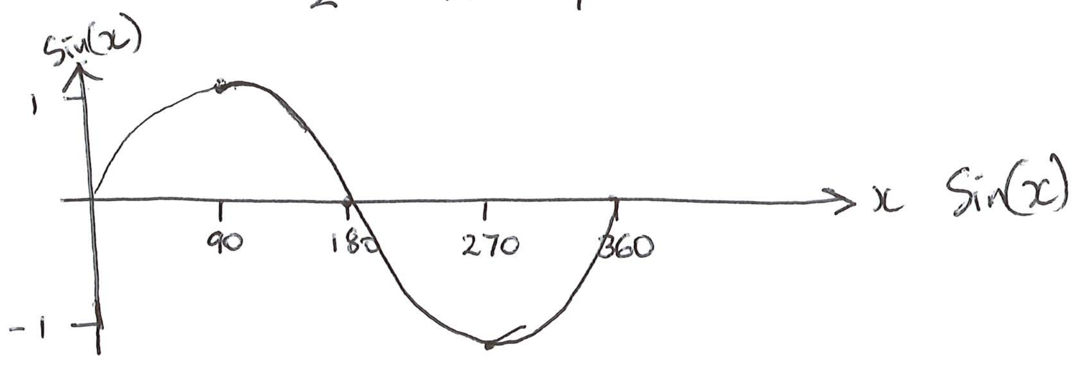
(18) $\tan^2(45) = \frac{1}{1} \times \frac{1}{1} = 1$

(19) $\sin^2(45) = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

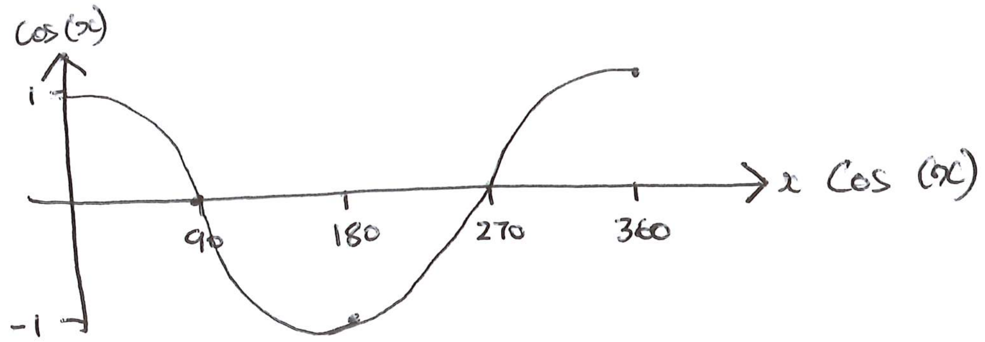
(20) $\tan^2(60) = \frac{\sqrt{3}}{1} \times \frac{\sqrt{3}}{1} = 3$

(21) $\sin^2(60) = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}$

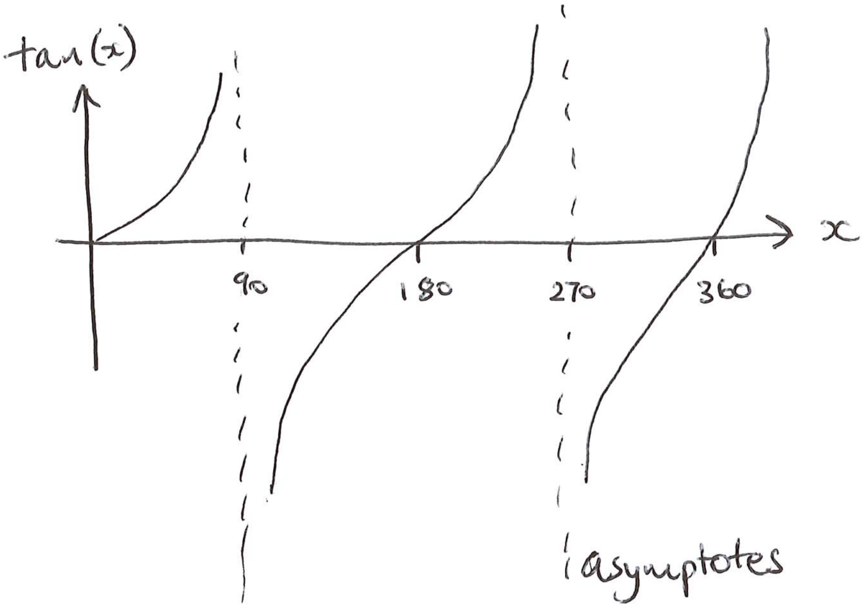
(22)



(23)



(24)



---

(25)

	$6x$	$+a$
$4x$	$24x^2$	$+4ax$
-4	$-24x$	$-4a$ or b

$$-24x + 4ax = -12x$$

$$4ax = 24x - 12x$$

$$= 12x$$

$$\therefore a = \frac{12x}{4x}$$

$$\boxed{\therefore a = 3}$$

$$b = -4a$$

$$= -4(3)$$

$$\boxed{\therefore b = -12}$$

(26)

	$3x$	$+ a$
$7x$	$21x^2$	$+ 7ax$
-5	$-15x$	$-5a = b$

$7ax - 15x = -x$

$7ax = 15x - x$

$\therefore 7ax = 14x$

$\therefore a = \frac{14x}{7x}$

$\boxed{\therefore a = 2}$

As $b = -5a$

$\boxed{b = -10}$

(27)

	$8x$	$-a$
$5x$	$40x^2$	$-5ax$
-2	$-16x$	$+2a = b$

$-16x - 5ax = -19x$

$-5ax = -19x + 16x$

$= -3x$

$\therefore a = \frac{-3x}{-5x}$

$\boxed{a = \frac{3}{5}}$

As $b = 2a$, $\boxed{b = \frac{3}{5} \times \frac{2}{1} = \frac{6}{5}}$

(28)

	$3x$	$+4$
$9x$	$27x^2$	$+36x$
$-a$	$-3ax$	$-4a = b$

$$36x - 3ax = 0$$

$$-3ax = -36x$$

$$a = \frac{-36x}{-3x}$$

$$\boxed{a = 12}$$

As $b = -4a$, $b = -4(12)$

$$= -48$$

So $\boxed{b = -48}$

---

(29)

	ax	$+5$
$5x$	$5ax^2$	$+25x$
$+b$	$+abx$	$+5b = c$

$$5ax^2 = 35x^2$$

$$\therefore a = \frac{35x^2}{5x^2}$$

$$\boxed{\therefore a = 7}$$

$$25x + abx = 46x$$

$$abx = 46x - 25x$$

$$= 21x$$

Substitute $a = 7$

$$7bx = 21x$$

$$\therefore b = \frac{21x}{7x}$$

$$\boxed{\therefore b = 3}$$

As $c = 5b$, $c = 5 \times 3$

$$\boxed{\therefore c = 15}$$

(30) If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$

$$y = 3x + 12 - x^2$$

$$\therefore \frac{dy}{dx} = 3 - 2x$$

---

(31) $y = 3x^3\left(x^2 + 5 - \frac{7}{x}\right)$

$$= 3x^5 + 15x^3 - \frac{21x^3}{x}$$

$$= 3x^5 + 15x^3 - 21x^2$$

$$\therefore \frac{dy}{dx} = 15x^4 + 45x^2 - 42x$$

---

(32) $y = (8x^3 + 5)(7x^9 - 3x)$

	$8x^3$	$+5$
$7x^9$	$56x^{12}$	$+35x^9$
$-3x$	$-24x^4$	$-15x$

$$\therefore y = 56x^{12} + 35x^9 - 24x^4 - 15x$$

$$\therefore \frac{dy}{dx} = 672x^{11} + 315x^8 - 96x^3 - 15$$

---

(33) $y = 5x^4\left(\frac{2}{x^3} + 4x^3\right)$

$$= \frac{10x^4}{x^3} + 20x^{12}$$

$$= 10x + 20x^{12}$$

$$\therefore \frac{dy}{dx} = 10\cancel{x} + \cancel{20}\ 240x^{11}$$

(34) $$y = 8x^5\left(\frac{4}{5x} - x + \frac{2}{3}\right)$$

$$= \frac{32x^5}{5x} - 8x^6 + \frac{16x^5}{3}$$

$$= \frac{32x^4}{5} - 8x^6 + \frac{16}{3}x^5$$

$$\therefore \frac{dy}{dx} = \frac{128}{5}x^3 - 48x^5 + \frac{80}{3}x^4$$

---

(35) $$y = \frac{3x - 12 + x^5}{x}$$

$$= \frac{3x}{x} - \frac{12}{x} + \frac{x^5}{x}$$

$$= 3 - 12x^{-1} + x^4$$

$$\therefore \frac{dy}{dx} = 12x^{-2} + 4x^3$$

---

(36) $$\begin{pmatrix} -2 & 3 \\ -4 & 0 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 5 & 9 \end{pmatrix} = \begin{pmatrix} -2\times4 + 3\times5 & -2\times3 + 3\times9 \\ -4\times4 + 0\times5 & -4\times3 + 0\times9 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 21 \\ 16 & -12 \end{pmatrix}$$

---

(37) $$\begin{pmatrix} 4 & 3 \\ 5 & 9 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} 4\times-2 + 3\times-4 & 4\times3 + 3\times0 \\ 5\times-2 + 9\times-4 & 5\times3 + 9\times0 \end{pmatrix} = \begin{pmatrix} -20 & 12 \\ -46 & 15 \end{pmatrix}$$

(38) $\begin{pmatrix} -2 & 3 \\ -4 & 0 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} -2\times -2 + 3\times -4 & -2\times 3 + 3\times 0 \\ -4\times -2 + 0\times -4 & -4\times 3 + 0\times 0 \end{pmatrix} = \begin{pmatrix} -16 & -6 \\ 8 & -12 \end{pmatrix}$

---

(39) $\begin{pmatrix} 4 & 3 \\ 5 & 9 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 5 & 9 \end{pmatrix} = \begin{pmatrix} 4\times 4 + 3\times 5 & 4\times 3 + 3\times 9 \\ 5\times 4 + 9\times 5 & 5\times 3 + 9\times 9 \end{pmatrix} = \begin{pmatrix} 31 & 39 \\ 65 & 96 \end{pmatrix}$

---

(40) $\begin{pmatrix} \frac{3}{5} & 1 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 5 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{5}\times 3 + 1\times 5 & \frac{3}{5}\times 4 + 1\times -\frac{1}{2} \\ -5\times 3 + 2\times 5 & -5\times 4 + 2\times -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 6\frac{4}{5} & 1\frac{9}{10} \\ -5 & -21 \end{pmatrix}$

---

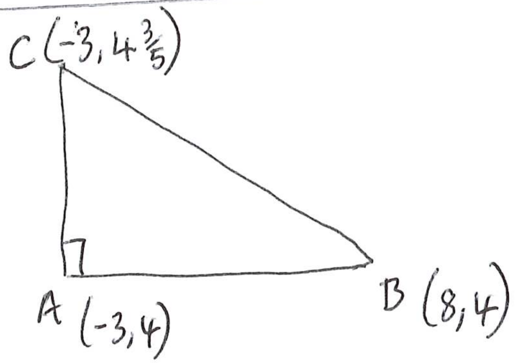
(41) $\begin{pmatrix} \frac{2}{3} & \frac{4}{5} \\ -\frac{1}{3} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}\times 1 + \frac{4}{5}\times 2 & \frac{2}{3}\times 1 + \frac{4}{5}\times 0 \\ -\frac{1}{3}\times 1 + \frac{1}{4}\times 2 & -\frac{1}{3}\times 1 + \frac{1}{4}\times 0 \end{pmatrix} = \begin{pmatrix} \frac{8}{5} + \frac{2}{3} & \frac{2}{3} \\ \frac{1}{2} - \frac{1}{3} & -\frac{1}{3} \end{pmatrix}$

$= \begin{pmatrix} \frac{34}{15} & \frac{2}{3} \\ \frac{1}{6} & -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 2\frac{4}{15} & \frac{2}{3} \\ \frac{1}{6} & -\frac{1}{3} \end{pmatrix}$

$\frac{24}{15} + \frac{10}{15} = \frac{34}{15}$

---

(42)



$AB = 3 + 8$

$= 11$

$AC = 4\frac{3}{5} - 4$

$= \frac{3}{5} = 0.6$

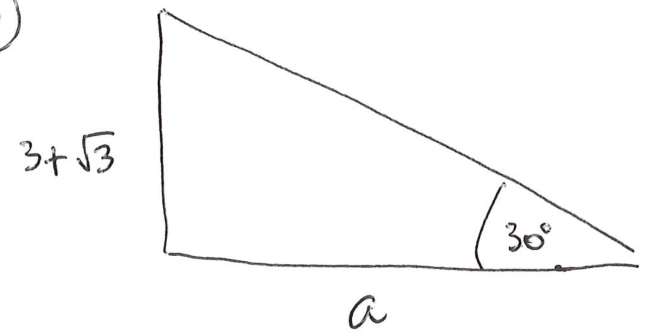
$BC = \sqrt{121 + 0.36}$

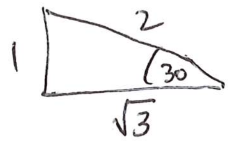
$= \sqrt{121.36}$

$= 11.01635148$

≈ 11.02 to 4 sig fig.

(43)





$$\tan 30 = \frac{1}{\sqrt{3}}$$

$$\tan 30 = \frac{3+\sqrt{3}}{a}$$

$$\therefore \quad a \tan 30 = 3+\sqrt{3}$$

$$a = \frac{3+\sqrt{3}}{\tan 30}$$

$$= \frac{3+\sqrt{3}}{\frac{1}{\sqrt{3}}}$$

$$= 3\sqrt{3}+3$$

Area of a triangle $= \frac{1}{2}bh$

$$= \frac{1}{2}(3+\sqrt{3})(3\sqrt{3}+3)$$

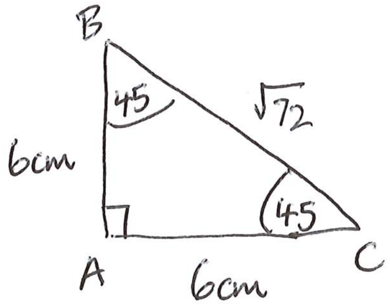
$$= \frac{1}{2}(12+12\sqrt{3})$$

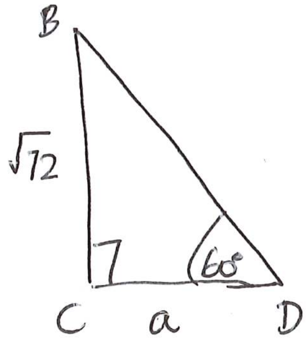
$$= 6+6\sqrt{3}$$

$$= 6(1+\sqrt{3})$$

	3	$+\sqrt{3}$
$3\sqrt{3}$	$9\sqrt{3}$	$+9$
$+\sqrt{3}$	$+3\sqrt{3}$	$+3$

(44)





$$BC = \sqrt{36+36}$$
$$= \sqrt{72}.$$

$$\text{Tan } 60 = \frac{O}{A} = \frac{\sqrt{72}}{a}$$

$$\therefore \quad a = \frac{\sqrt{72}}{\text{Tan } 60}$$
$$= \frac{\sqrt{72}}{\sqrt{3}}$$
$$= \frac{\sqrt{3}\sqrt{4}\sqrt{6}}{\sqrt{3}}$$
$$= \sqrt{4}\sqrt{6}$$
$$= 2\sqrt{6} \text{ as required.}$$

---

(45)

$-3 \quad 14 \quad 39 \quad 72$

$+17 \quad +25 \quad +33$

$+8 \quad +8$

$$n^2 \text{ co-efficient} = \frac{8}{2} = 4$$

$$\boxed{4n^2 + 5n - 12}$$

n	1	2	3	4
$4n^2$	4	16	36	64
	-7	-2	3	8

$+5 \quad +5 \quad +5 \quad \Rightarrow +5n$

n	1	2	3	4
$5n$	5	10	15	20
	-7	-2	3	8
-12				

$\Rightarrow -12$

(46) $2n^2 + 10n < 3000$

$\therefore 2n^2 + 10n - 3000 < 0$

$$n < \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore n < \frac{-10 \pm \sqrt{100 - 4(2)(-3000)}}{4}$$

$$= \frac{-10 \pm \sqrt{24100}}{4}$$

$= \cancel{...}\ 36.22983346$

$\therefore \boxed{n \leqslant 36}$

---

(47) 6 5 2 −3 −10

−1 −3 −5 −7

−2 −2 −2

n^2 term coefficient $= \frac{-2}{2} = -1$

$\boxed{-n^2 + 2n + 5}$ nth term.

n	1	2	3	4	5
$-n^2$	−1	−4	−9	−16	−25
$t_n - n^2$	7	9	11	13	15

+2

n	1	2	3	4	5
$2n$	2	4	6	8	10

+5 to get

7 9 11 13 15

(48) 5 1 −7 −19

$$5 \xrightarrow{-4} 1 \xrightarrow{-8} -7 \xrightarrow{-12} -19$$

$$-4 \xrightarrow{-4} -8 \xrightarrow{-4} -12$$

n^2 coefficient $= \frac{-4}{2} = -2$

$$\boxed{-2n^2 + 2n + 5}$$

n	1	2	3	4
$-2n^2$	-2	-8	-18	-32
$t_n - 2n^2$	7	9	11	13

$-19-(-32)$

$$7 \xrightarrow{+2} 9 \xrightarrow{+2} 11 \xrightarrow{+2} 13 \Rightarrow +2n$$

2 4 6 8

$7-2=5 \quad \Rightarrow +5$

---

(49) $3n^2 - 4n < 600$

$\therefore \; 3n^2 - 4n - 600 < 0$

Using the quadratic formula

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+4 \pm \sqrt{16 - 4(3)(-600)}}{6}$$

$$= \frac{+4 \pm \sqrt{7216}}{6}$$

$$= 14.82$$

which means there are 14 terms lower than 600 in the sequence $3n^2 - 4n$.

(50)

$$\frac{\sqrt{5}}{5+\sqrt{5}} = \frac{\sqrt{5}}{5+\sqrt{5}} \times \frac{5-\sqrt{5}}{5-\sqrt{5}} = \frac{5\sqrt{5}-5}{20} = \frac{\sqrt{5}-1}{4}$$

	5	$+\sqrt{5}$
5	25	$+5\sqrt{5}$
$-\sqrt{5}$	$-5\sqrt{5}$	-5

	5	$-\sqrt{5}$
$\sqrt{5}$	$5\sqrt{5}$	-5

Remember to multiply by 1 in the form of the conjugate divided by the conjugate

(51)

$$\frac{\sqrt{7}}{7+\sqrt{7}} = \frac{\sqrt{7}}{7+\sqrt{7}} \times \frac{7-\sqrt{7}}{7-\sqrt{7}} = \frac{7\sqrt{7}-7}{42} = \frac{\sqrt{7}-1}{6}$$

	7	$+\sqrt{7}$
7	49	$+7\sqrt{7}$
$-\sqrt{7}$	$-7\sqrt{7}$	-7

	7	$-\sqrt{7}$
$\sqrt{7}$	$7\sqrt{7}$	-7

(52)

$$\frac{\sqrt{13}}{13+\sqrt{13}} = \frac{\sqrt{13}}{13+\sqrt{13}} \times \frac{13-\sqrt{13}}{13-\sqrt{13}} = \frac{13\sqrt{13}-13}{156} = \frac{\sqrt{13}-1}{12}$$

	13	$+\sqrt{13}$
13	169	$+13\sqrt{13}$
$-\sqrt{13}$	$-13\sqrt{13}$	-13

	13	$-\sqrt{13}$
$\sqrt{13}$	$13\sqrt{13}$	-13

53 Pascals Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$(a+b)^5 = 1a^5b^0 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$a = 5$, $b = 3x$

$$(5+3x)^5 = (5)^5 + 5(5)^4(3x)^1 + 10(5)^3(3x)^2 + 10(5)^2(3x)^3 + 5(5)^1(3x)^4 + (3x)^5$$

$$= 3125 + 9375x + 11250x^2 + 6750x^3 + 2025x^4 + 243x^5$$

54

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$(a+b)^4 = 1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$$

$$(2+5x)^4 = (2)^4 + 4(2)^3(5x)^1 + 6(2)^2(5x)^2 + 4(2)^1(5x)^3 + 1(5x)^4$$

$$= 16 + 160x + 600x^2 + 1000x^3 + 625x^4$$

(55)

$$\begin{array}{c} 1 \\ 1\ 1 \\ 1\ 2\ 1 \\ 1\ 3\ 3\ 1 \\ 1\ 4\ 6\ 4\ 1 \\ 1\ 5\ 10\ 10\ 5\ 1 \end{array}$$

$$4\sqrt{x} = 4x^{\frac{1}{2}}$$

$$(a+b)^5 = 1a^5b^0 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1a^0b^5$$

$$(9+4\sqrt{x})^5 = 9^5 + 5(9)^4(4x^{\frac{1}{2}})^1 + 10(9)^3(4x^{\frac{1}{2}})^2 + 10(9)^2(4x^{\frac{1}{2}})^3 + 5(9)^1(4x^{\frac{1}{2}})^4 + (4x^{\frac{1}{2}})^5$$

$$= 59{,}049 + 131{,}220x^{\frac{1}{2}} + 116{,}640x + 51{,}840x^{3/2} + 11{,}520x^2 + 1024x^{\frac{5}{2}}$$

---

(56)

$$\frac{5n^2}{n^2+3} = \frac{15}{4}$$

$$\therefore\ 4(5n^2) = 15(n^2+3)$$

$$\therefore\ 20n^2 = 15n^2 + 45$$

$$\therefore\ 5n^2 = 45$$

$$\therefore\ n^2 = 9$$

$$\therefore\ n = 3$$

---

(57)

$$\frac{5n^2}{n^2+3} = \frac{\frac{5n^2}{n^2}}{\frac{n^2}{n^2} + \frac{3}{n^2}} = \frac{5}{1+\frac{3}{n^2}}$$

As $n \to \infty$, $\frac{3}{n^2} \to 0$ So $\frac{5}{1+\frac{3}{n^2}} \to \frac{5}{1} = 5$

(58)

$$\frac{3n^2}{n^2+5} = \frac{\frac{3n^2}{n^2}}{\frac{n^2}{n^2}+\frac{5}{n^2}} = \frac{3}{1+\frac{5}{n^2}}$$

As $n \to \infty$, $\frac{5}{n^2} \to 0$ and $\frac{3}{1+\frac{5}{n^2}} \to \frac{3}{1} = 3$

---

(59)

$$de = \frac{7d+9}{4}$$

$$4de = 7d + 9$$

$$4de - 7d = 9$$

$$d(4e-7) = 9$$

$$\therefore \quad d = \frac{9}{4e-7}$$

---

(60)

$$gh = \frac{6h+9}{12}$$

$$12gh = 6h + 9$$

$$12gh - 6h = 9$$

$$h(12g-6) = 9$$

$$h = \frac{9}{12g-6}$$

$$= \frac{3}{4g-2}$$

(61) $8x^2 + 32x - 19 = 8\left[x^2 + 4x\right] - 19$

$= 8\left[(x+2)^2 - 4\right] - 19$

$= 8(x+2)^2 - 32 - 19$

$= 8(x+2)^2 - 51$

---

(62) $5x^2 - 25x + 17 = 5\left[x^2 - 5x\right] + 17$

$= 5\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 17$

$= 5\left(x - \frac{5}{2}\right)^2 - \frac{125}{4} + 17$

$= 5\left(x - \frac{5}{2}\right)^2 - \frac{57}{4}$

---

(63) $12x^2 - 30x + 45 = 12\left[x^2 - \frac{5}{2}x\right] + 45$

$= 12\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] + 45$

$= 12\left(x - \frac{5}{4}\right)^2 - \frac{300}{16} + 45$

$= 12\left(x - \frac{5}{4}\right)^2 + \frac{105}{4}$

(64) $y = 2x^3 - 3x^2 - 12x + 4$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

When $\frac{dy}{dx} = 0$, $6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore 6(x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

When $x = -1$, $y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 4$

$= (-2) - (3) - (-12) + 4$

$= 11$

So one stationary point is at $(-1, 11)$

When $x = 2$, $y = 2(2)^3 - 3(2)^2 - 12(2) + 4$

$= (16) - (12) - (24) + 4$

$= -16$

So the other stationary point is at $(2, -16)$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

$\frac{d^2y}{dx^2} = 12x - 6$

At point $(-1, 11)$, $x = -1$ so $\frac{d^2y}{dx^2} = 12(-1) - 6 = -18 < 0$

so $(-1, 11)$ is a maximum point.

At point $(2, -16)$, $x = 2$ so $\frac{d^2y}{dx^2} = 12(2) - 6 = +18 > 0$

so $(2, -16)$ is a minimum point.

(65) $y = x^3 - 12x + 3$

$$\frac{dy}{dx} = 3x^2 - 12$$

When $\frac{dy}{dx} = 0$, $3x^2 - 12 = 0$

$\therefore \quad 3(x^2 - 4) = 0$

$\therefore \quad 3(x+2)(x-2) = 0$

$\therefore \quad x = -2$ and $x = 2$ are at stationary points.

When $x = -2$, $y = (-2)^3 - 12(-2) + 3$

$= -8 + 24 + 3$

$= 19$

So the stationary point is at $(-2, 19)$.

When $x = 2$, $y = (2)^3 - 12(2) + 3$

$= 8 - 24 + 3$

$= -13$

So the stationary point is at $(2, -13)$.

~~When~~ $\frac{d^2y}{dx^2} = 6x$ (Just differentiate $3x^2 - 12$ again)

When $x = -2$, $\frac{d^2y}{dx^2} = -12 < 0$ so $(-2, 19)$ is a maximum.

When $x = 2$, $\frac{d^2y}{dx^2} = 12 > 0$ so $(2, -13)$ is a minimum.

(66) $y = x^3 - 4x^2 + 9$

$$\frac{dy}{dx} = 3x^2 - 8x$$

$$\Rightarrow x = 0 \text{ or } x = \frac{8}{3}$$

$$\frac{d^2y}{dx^2} = 6x - 8$$

Find the co-ordinates of the stationary points

$x = 0$, $y = 0^3 - 4(0)^2 + 9 = 9$ so at $(0,9)$

$$x = \frac{8}{3}, \quad y = \left(\frac{8}{3}\right)^3 - 4\left(\frac{8}{3}\right)^2 + 9 = \frac{512}{27} - 4\left(\frac{64}{9}\right) + 9$$

$$= \frac{512}{27} - 4\left(\frac{192}{27}\right) + \frac{243}{27}$$

$$= \frac{512}{27} - \frac{768}{27} + \frac{243}{27}$$

$$= -\frac{13}{27}$$

So stationary points are at $(0,9)$ and $\left(\frac{8}{3}, -\frac{13}{27}\right)$.

When $x = 0$ at $(0,9)$, $6x - 8 = -8 < 0$ maximum.

When $x = \frac{8}{3}$ at $\left(\frac{8}{3}, -\frac{13}{27}\right)$ $6x - 8 = +8 > 0$ minimum.